Bremsstrahlung and nonlinear currents in a dense plasma exposed to an intense laser field

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The classical kinetic equation for electrons in a dense strongly coupled plasma, interacting with laser radiation, is considered. The effect of strong ion-ion coupling on nonlinear bremsstrahlung and higher-order harmonics of the electric current is investigated. Particular consideration is devoted to a strongly degenerate plasma. Analytical formulas for nonlinear conductivities versus field intensity are derived for this case. These formulas are valid in the nonasymptotic region $0<\varepsilon_E<\varepsilon_F$ (ε_E is the electron quiver energy in a laser field, ε_F is the electron Fermi energy in metals).

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I. INTRODUCTION

Absorption by inverse nonlinear bremsstrahlung was intensively studied during the last three decades [1–11]. First considered in [1–7], this problem is still in the focus of current research [8–11]. The physical picture of inverse bremsstrahlung absorption in a super-intense laser field is rather simple. Qualitative estimates can be obtained by taking the conventional formula for the bremsstrahlung absorption and substituting the electron quiver velocity v_E due to a laser field for the characteristic electron velocity v_e in the absence of a field. Thus the problem for super-high field intensities mainly consists in obtaining correct numerical and logarithmic factors. In some preceding papers on the nonlinear bremsstrahlung in a gaseous plasma, see, e.g., [6,7], these factors, which can be large in magnitude, have been lost. The most complete asymptotic analysis of the problem of the bremsstrahlung absorption in gaseous plasmas has been given recently [10]. However, some important questions were not considered.

First, density effects (plasma coupling effects) relevant for nonlinear inverse bremsstrahlung have not been considered in the above references. This question is of particular interest in connection with the recent investigation of ultrashort laser pulses interacting with solids [12–15]. Under such conditions, a very dense plasma with unusual properties is created: the electron subsystem is heated, while the ion subsystem remains cool and therefore strongly coupled (correlated). As is well known, ion-ion correlations strongly influence electron-ion Coulomb scattering with momentum transfer $p<p_a\sim\hbar/a$, where a is the correlation length in the ion subsystem, the radius of the ion sphere, $4\pi a^3/3=1/n_i$, where n_i is the ion number density. In a dense plasma p_a can be come larger than the characteristic electron momentum mv_e and therefore ion-ion correlations are important. This case will be, in particular, considered in the present paper.

Secondly, the question of higher-order harmonics of the electric current in a super-intense field has not been discussed in the literature since the pioneering paper [1]. However, in [1] the double logarithmic factors were estimated in a rather approximate manner, and only a gaseous plasma was considered. Therefore this question is worth investigating in more detail, including effects of strong ion-ion correlations, and we shall consider it in the present paper as well.

Thirdly, the problems discussed above refer to the asymptotic case $v_E\gg v_e$ and thus they reduce to the question of correct logarithmic factors. Thus far the nonasymptotic region $v_e\sim v_E$ has not been considered in the literature. However, the case of modest intensities $v_E\le v_e$ is of practical interest relevant to laser radiation interaction with condensed matter. Indeed, for field intensities as high as $I\sim10^{15}$ W/cm^2 and photon energies $\hbar\omega\sim4$ eV, typical for recent experiments [12], the quiver energy $mv_E^2/2\sim10$ eV is comparable with the electron Fermi energy in solids. At the same time the electron subsystem, even when heated up to several eV, remains strongly degenerate. Thus in the present paper we shall consider bremsstrahlung and higher-order harmonics of the electric current in the nonasymptotic range $v_E\sim v_e$ for the case of strongly coupled, strongly degenerate dense plasma. It is important that the results obtained for nonlinear conductivities can be presented in a purely analytical form over the entire range ($0<v_E<v_e$).

II. THE ELECTRON KINETIC EQUATION

The kinetic equation for the electron distribution function $f(\mathbf{p},t)$ of a strongly correlated plasma exposed to a high-frequency electrical field $\mathbf{E}\cos\omega t$ can be expressed in the form (hereinafter we assume the Planck constant $\hbar=1$) [16]

$$\frac{\partial f(\mathbf{p},t)}{\partial t}+e\mathbf{E}\cos(\omega t)\frac{\partial f}{\partial \mathbf{p}}=n_i\int\frac{d^3q}{(2\pi)^3}(f(\mathbf{p}+\mathbf{q},t)-f(\mathbf{p},t))2\pi\delta(\varepsilon(\mathbf{p}+\mathbf{q})-\varepsilon(\mathbf{p}))|V_q|^2S(q)\,,$$

$$|V_q|^2=\frac{(4\pi e^2Z)^2}{q^4}\,. \quad (1)$$

Here, $\varepsilon(p)=p^2/2m$ is the electron kinetic energy, m,e are the electron mass and charge, respectively, $|Ze|$ is the ion charge, and $S(q)$ is the ion structure factor, which describes the correlation in ion positions and relates to the Fourier transformation of the two-particle distribution function [16].

An analogous approach was chosen in [11]. However, the authors of [11] used the collisional integral in the Landau form, as in [1], which is not adequate for treating logarithmic factors as well as ion-ion correlation effects. The authors of [11] were aiming for a different goal and solved the kinetic equation averaged over fast oscillations. This limit is not suitable to consider the conductivity problem. Lastly, the question of the applicability of the classical kinetic equation (1) to describe the electron–laser-field interaction has not been considered in [11]. We shall return to this question in Sec. VI.

We introduce a function F which represents the electron distribution in the oscillating frame of reference:

$$F(\mathbf{P},t)=f(\mathbf{P}+(e\mathbf{E}/\omega)\sin\omega t,t)\ . \tag{2}$$

Thus we can rewrite Eq. (1) in a form convenient for further analysis:

$$\frac{\partial F}{\partial t}=n_i\int\frac{d^3q}{(2\pi)^3}|V_q|^2S(q)[F(\mathbf{P}+\mathbf{q})-F(\mathbf{P})]2\pi\delta\left[\varepsilon(\mathbf{P}+\mathbf{q})-\varepsilon(\mathbf{P})+\frac{e\mathbf{E}\cdot\mathbf{q}\sin\omega t}{m\omega}\right]. \tag{3}$$

Let us seek the solution of this equation in the form

$$F(\mathbf{P},t)=\sum_{n=-\infty}\exp(in\omega t)\phi_n(\mathbf{P},t)\ . \tag{4}$$

After substitution of Eq. (4) into Eq. (3) we get

$$\begin{aligned}in\omega\phi_n(\mathbf{P},t)+\frac{\partial}{\partial t}\phi_n(\mathbf{P},t)=n_i\int\frac{d^3q}{(2\pi)^3}S(q)|V_q|^2\int_{-\infty}^{\infty}dx\sum_{n'}J_{n-n'}\left[-x\frac{e\mathbf{E}\cdot\mathbf{q}}{m\omega}\right]\exp\{ix[\varepsilon(\mathbf{P})-\varepsilon(\mathbf{P}+\mathbf{q})]\}\\ \times[\phi_{n'}(\mathbf{P}+\mathbf{q},t)-\phi_{n'}(\mathbf{P},t)]\ ,\end{aligned} \tag{5}$$

where J_n are Bessel functions.

We consider the case when the ω is much larger than the electron-ion collision frequency. It means that the functions ϕ_n vary very slowly during the time $1/\omega$. For $n\neq0$, we can neglect the term with the time derivative on the left-hand side of Eq. (5) and omit terms with $n'\neq0$ on the right-hand side. Then each harmonic ϕ_n can be expressed in terms of ϕ_0:

$$\phi_n(\mathbf{P},t)=\frac{n_i}{in\omega}\int\frac{d^3q}{(2\pi)^3}S(q)|V_q|^2\int_{-\infty}^{\infty}dx\,J_n\left[-x\frac{e\mathbf{E}\cdot\mathbf{q}}{m\omega}\right]\exp\{ix[\varepsilon(\mathbf{P})-\varepsilon(\mathbf{P}+\mathbf{q})]\}[\phi_0(\mathbf{P}+\mathbf{q},t)-\phi_0(\mathbf{P},t)]\ . \tag{6}$$

Using Eq. (6) one can directly verify that the following identities hold for $n\neq0$:

$$\int\phi_n(\mathbf{P},t)\frac{d^3P}{(2\pi)^3}=0 \tag{7}$$

and thus the ϕ_0 satisfies the conventional normalization condition:

$$\int\phi_0(\mathbf{P},t)\frac{d^3P}{(2\pi)^3}=n_e\ , \tag{8}$$

where n_e is the electron number density. The equation for ϕ_0 was investigated in [8,9,11]. The time dependence of ϕ_0 is attributed to the heating of the electron subsystem. In the present paper, we shall mainly concentrate on harmonics ϕ_n, $n\neq0$ and a particular form of ϕ_0 is not important for this purpose. We assume only that the ϕ_0 satisfies the general condition

$$\phi_0(\mathbf{P})=\phi_0(-\mathbf{P})\ , \tag{9}$$

valid for a medium possessing inversion symmetry.

III. NONLINEAR CURRENTS

The electron current $\mathbf{j}'$ of a plasma can be calculated from the following definition:

$$\begin{aligned}\mathbf{j}'&=e\int\frac{d^3p}{(2\pi)^3}f(\mathbf{p},t)\frac{\mathbf{p}}{m}\\&=e\int\frac{d^3P}{(2\pi)^3}F(\mathbf{P},t)\frac{\mathbf{P}+(\mathbf{E}e/\omega)\sin\omega t}{m}\ .\end{aligned} \tag{10}$$

Accordingly the zero-order harmonic can be written as

$$\mathbf{j}_0' = e\int \frac{d^3P}{(2\pi)^3}\frac{\mathbf{P}}{m}\phi_0(\mathbf{P},t) + \frac{\mathbf{E}e^2}{m\omega}n_e \sin\omega t \ . \tag{11}$$

Due to Eq. (9), the first integral in Eq. (11) vanishes and only the second term containing **E** survives. However if $\mathbf{j}_n'$ with $n\neq 0$ are considered, we have the opposite situation, and due to Eq. (7) the term with **E** does not contribute. Finally substituting Eq. (6) into Eq. (10) we obtain (for $n\neq 0$)

$$\mathbf{j}_n' = \mathbf{e}_E \exp(in\omega t)\frac{n_i e}{in\omega}\int \frac{d^3P}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3} S(q)|V_q|^2 \frac{\mathbf{q}\cdot\mathbf{e}_E}{m} \times \int_{-\infty}^{\infty} dx\, J_n\left[-x\frac{e\mathbf{E}\cdot\mathbf{q}}{m\omega}\right]\exp\{ix[\varepsilon(\mathbf{P})-\varepsilon(\mathbf{P}+\mathbf{q})]\}\phi_0(\mathbf{P},t)\ , \tag{12}$$

where $\mathbf{e}_E$ is the unit vector in the **E** direction. It can be shown that due to Eq. (9) $\mathbf{j}_n'$ exactly vanishes for even n, as one should expect for a medium with inversion symmetry. Therefore in this case only odd harmonics of the electric current can be generated, and hereinafter n is assumed to be odd.

The integral with respect to x in Eq. (12) can be calculated, resulting in

$$\mathbf{j}_n' = \mathbf{e}_E \exp(in\omega t)\frac{n_i e}{n\omega}\int \frac{d^3P}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3} S(q)|V_q|^2 \frac{\mathbf{q}\cdot\mathbf{e}_E}{m}\phi_0(\mathbf{P})\frac{2}{\{(e\mathbf{E}\cdot\mathbf{q})^2/(m\omega)^2-[\varepsilon(\mathbf{P})-\varepsilon(\mathbf{P}+\mathbf{q})]^2\}^{1/2}} \times \sin\left[n \arcsin\frac{\varepsilon(\mathbf{P})-\varepsilon(\mathbf{P}+\mathbf{q})}{-e\mathbf{E}\cdot\mathbf{q}/(m\omega)}\right]\ , \tag{13}$$

where the actual region of integration over **p** and **q** coincides with the region of the definition of the integrand.

IV. THE INTENSE-LASER-FIELD LIMIT

For concrete calculations, we chose a particular form of the $S(q)$ obtained for the one-component plasma model [17], which is characterized by similarity coupling parameter $\Gamma = Z^2e^2/aT$ (T is the ion temperature). Figure 1 depicts the behavior of $S(q)$ at $\Gamma=120$, which is a typical value for strongly coupled plasmas of the liquid-metal type we are interested in here. It is seen that $S(q)$ can be approximated by a discontinuous function represented as the combination of a parabola Cq^2 and a steplike function joined together at a certain point $q=\kappa\sim 5/a$. In the case of a strongly degenerate plasma, the electron momentum is typically of the order of the average momentum for the Fermi distribution: $P_e\sim 1.4Z^{1/3}/a$. Therefore if Z is not too large, one has $\kappa > P_e$, and the ion-ion correlation strongly affects electron-ion scattering events. In this case we can split the integration region with respect to $|\mathbf{q}|$ in (13) into two parts: $[0,\kappa]$ and $[\kappa,\infty]$. We consider the calculation of the respective contributions j_0' and j_∞' separately. In the latter one the P dependence of the integrand in Eq. (13) can be neglected because $P_e<q$ in this domain. As we already mentioned above, $S(q)=1$ in this region. After implementing integrations over **P** and the directions of **q** we obtain

$$\mathbf{j}_{n,\infty}' = -\mathbf{e}_E n_i n_e \exp(in\omega t)\frac{\omega}{neE^2}\frac{1}{2\pi^2}(4\pi e^2 Z)^2 J_n'\ ,$$

$$J_n' = \int_\kappa^\infty \frac{dq}{q}\int_\Delta^{\pi/2}\frac{\sin ny}{\sin^2 y}dy\ , \tag{14}$$

where $\Delta=\arcsin(q\omega/2|eE|)$ and the J_n' in Eq. (14) should be considered to be zero if $q\omega/2|eE|\geq 1$, or, equivalently, the region of the integration with respect to q coincides with the region of the definition of the function Δ. The integral J_n', after integration by parts with respect to q, results in

$$J_n' = \ln A\int_{1/A}^{1}\frac{dz}{z\sqrt{1-z^2}}\frac{\sin(n\arcsin z)}{z} - \int_{1/A}^{1}\ln z\frac{dz}{z\sqrt{1-z^2}}\frac{\sin(n\arcsin z)}{z}\ , \tag{15}$$

where $A=|2eE/\omega\kappa|$. Thus far we have not used the fact that $A\gg 1$ (strong fields). Both integrals in Eq. (15) diverge at the low limit if $A\to\infty$ and converge at the upper limit. Therefore seeking for the asymptotic behavior at $A\to\infty$ we can substitute the upper limit by an arbitrary quantity $z_0<1$, which does not affect the final result. Then integration by parts in Eq. (15) is possible and one obtains under the condition $n<A$

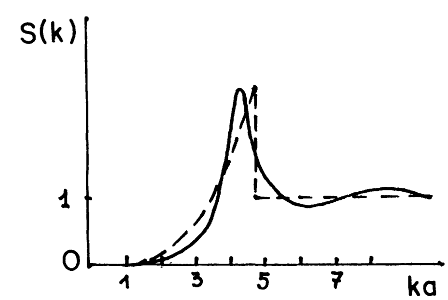

FIG. 1. Typical behavior of the ion-ion structure factor $S(q)$ and its approximation. The plot corresponds to $\Gamma=120$ [17].

$$J_n' = \frac{n}{2}\left[\ln\frac{|2eE|}{\omega\kappa}\right]^2 . \tag{16}$$

For $n > A$ the integrals J_n' are comparatively small due to rapid oscillations of the integrand in Eq. (15). Thus from Eqs. (16) and (14), we have for $n < A$

$$\mathbf{j}_{n,\infty}' = -\mathbf{e}_{\mathrm{E}} n_i n_e \exp(in\omega t)\frac{\omega}{E^2}4e^3Z^2\left[\ln\frac{|2eE|}{\omega\kappa}\right]^2 \tag{17}$$

and harmonics with $n > A$ can be neglected.

Let us consider now the contribution from the region $0 \le q \le \kappa$ to Eq. (13). In this case we can use the parabolic approximation for $S(q)$:

$$S(q) = S_m\left[\frac{q}{\kappa}\right]^2 , \quad q \le \kappa \tag{18}$$

where S_m is the altitude of the first maximum of the structure factor. Separating the integration with respect to directions of $\mathbf{q}$ and introducing the new variable $x = |eE|\cos\alpha|q\omega$, where α is the angle between $\mathbf{q}$ and $\mathbf{E}$ we obtain

$$\mathbf{j}_{n,0}' = -\mathbf{e}_{\mathrm{E}}\exp(in\omega t)\frac{n_i e}{n\omega} \times \int \frac{d^3P}{(2\pi)^6}\phi_0(P)\int_0^\kappa dq\, q^2|V_q|^2S(q)\frac{2q}{m}2\pi J_n'' ,$$

$$J_n'' = \frac{2m\omega^2}{e^2E^2}\int_0^{|eE|/q\omega} x\,dx\frac{1}{\sqrt{x^2-\Delta_1^2}}\sin\left[n\arcsin\frac{\Delta_1}{x}\right] , \tag{19}$$

$$\Delta_1 = \frac{m}{q^2}[\varepsilon(\mathbf{P}+\mathbf{q}) - \varepsilon(\mathbf{P})] .$$

In the region of the integration in Eq. (19) we have $|eE|/\omega q > A/2 \gg 1$. For $A \to \infty$, the J_n'' logarithmically diverges at the upper limit. Therefore the integral J_n'' is determined mainly by the vicinity of its upper limit and we can substitute Δ_1 by the asymptotic expression $\frac{1}{2} + \mathbf{P}\cdot\mathbf{E}/qE$. Thus in the limit $A \to \infty$ one readily gets after straightforward integration

$$J_n'' = n\frac{2m\omega^2}{e^2E^2}\left[\frac{1}{2} + \frac{\mathbf{P}\cdot\mathbf{E}}{Eq}\right]\left[\ln\frac{|2eE|}{\omega q} - \ln\frac{|q+2\mathbf{P}\cdot\mathbf{E}/E|}{q}\right] . \tag{20}$$

Since $A \gg 1$, the first logarithm in the parentheses dominates the second one, except in a very narrow region in the vicinity of $q \sim 2\mathbf{P}\cdot\mathbf{E}/E$. However, the contribution of this region is small because of the compensating factor $(\frac{1}{2} + \mathbf{P}\cdot\mathbf{E}/Eq)$ vanishing at that point. Therefore the second logarithm Eq. (20) can be neglected and integrals in Eq. (19) can be immediately calculated:

$$\mathbf{j}_{n,0}' = -\mathbf{e}_{\mathrm{E}} n_i n_e \exp(in\omega t)\frac{\omega}{E^2}4e^3Z^2S_{\max}\ln\frac{2|eE|}{\omega\kappa} . \tag{21}$$

Again, Eq. (21) is valid for $n < A$, and harmonics with $n > A$ are small.

Let us introduce currents j_n and conductivities σ_n with $n = 2k-1$, $k \ge 1$ via the following definitions:

$$\mathbf{j}_n' = \mathbf{j}_{n,0}' + \mathbf{j}_{n,\infty}' ,$$
$$\mathbf{j}_n = \mathbf{j}_n' + \mathbf{j}_{-n}' = \sigma_n(\omega)\mathbf{E}\cos n\omega t . \tag{22}$$

Then from Eqs. (17), (21), and (22) one immediately gets

$$\sigma_n(\omega) = \frac{8|e|^3 n_e n_i Z^2\omega}{E^3}\ln\frac{|2eE|}{\omega\kappa}\left[\ln\frac{|2eE|}{\omega\kappa} + S_m\right] . \tag{23}$$

Let us emphasize at this point again that the conductivities $\sigma_n(\omega)$ do not depend upon n until $n < 2|2eE|/(\omega\kappa)$ and rapidly decrease for $n > |2eE|/(\omega\kappa)$. The relevance of this observation to the general high-order harmonic generation problem is briefly discussed in Sec. VI.

V. MODERATE LASER INTENSITIES

In order to consider the nonasymptotic region of moderate laser intensities, we have first of all to simplify Eq. (13). After the calculation of the integral with respect to directions of $\mathbf{P}$ in Eq. (13) we get

$$\mathbf{j}_n' = \mathbf{e}_E\exp(i\omega nt)\frac{4\pi n_i e}{\omega n^2}\int_0^\infty \frac{dP}{(2\pi)^3}P\phi_0(P)\int\frac{d^3q}{(2\pi)^3}\frac{\mathbf{q}\cdot\mathbf{e}_E}{q}\frac{e\mathbf{E}\cdot\mathbf{q}}{|e\mathbf{E}\cdot\mathbf{q}|}V_q|^2S(q) \times\left\{\cos\left[n\arcsin\left[\frac{q\omega(q-2P)}{2|e\mathbf{E}\cdot\mathbf{q}|}\right]\right] - \cos\left[n\arcsin\left[\frac{q\omega(q+2P)}{2|e\mathbf{E}\cdot\mathbf{q}|}\right]\right]\right\} , \tag{24}$$

where the actual ranges of integration over P and $\mathbf{q}$ coincide with the regions of the definition of the arcsin functions. These regions are different for the two corresponding terms in the integrand. Formula (24) is valid for all values of laser field intensity. In the preceding section, intense laser fields $v_E \gg v_e$ were considered. As we mentioned in the Introduction, radiation with $I\lambda^2 \sim 10^{14}$ W/cm^2 μm^2 can be considered as moderate, if one deals with the partially degenerated electron system in a solid metal. Thus in the present section we shall concentrate mainly on the case of strongly correlated plasma in fields of moderate intensity, when $v_e \sim v_F \sim v_E < \kappa/m$. Under such conditions, bearing in mind the above remark on the integration ranges, we can use the rough parabolic approximation (18) in the entire domain of integration in Eq. (24). It means we can set

$$|V_q|^2S(q) = \frac{B}{q^2}, \quad B = 4\pi e^4Z^2\frac{S_m}{\kappa^2} . \tag{25}$$

For the case of strong degeneracy, we have $\Phi_0(P) = 1$, $P < p_F = mv_F$, otherwise $\phi_0(P) = 0$. With these approximations the integral in Eq. (24) can be analytically calculated for each n. We present the results for the first and the third harmonics of the electric current:

$$\mathbf{j}_1=\sigma_0(\omega)\left[\frac{Ee}{\omega p_F}\right]^2\left[\left[\frac{\omega p_F}{|eE|}\right]^2-\frac{3}{20}\right]\mathbf{E}\cos\omega t\ ,$$
$$\mathbf{j}_3=\frac{1}{60}\sigma_0(\omega)\left[\frac{E_e}{\omega p_F}\right]^2\mathbf{E}\cos 3\omega t\ , \qquad (26)$$

where we introduced $\sigma_0(\omega)=12B\pi^3 n_i p_F^2 e^2/\omega^2$, the conductivity, in the high-frequency, weak-field limit. Formulas (26) are valid for $E<|\omega p_F/e|$.

VI. DISCUSSION OF THE RESULTS

First of all let us consider the conditions of the applicability of Eq. (1), in which the electron–laser-field interaction is described classically, but for the electron-ion scattering the quantum description is used. If we consider the time-averaged kinetic energy of an electron in a laser field before and after a single elastic electron-ion collision, then the time-averaged electron energy typically changes by $mv_E(v_e+v_E)$ [5]. This approximate formula joins together both weak- and strong-field limits. The classical description of the electron–laser-field interaction used in Eq. (1) is valid, if multiple photon processes dominate: $mv_E(v_e+v_E)\gg\hbar\omega$. This imposes limitations on the frequency and the intensity of the laser field. At the same time, the quantum description of the electron-ion interaction used in Eq. (1) is applicable if $\hbar(v_e+v_E)\geq e^2$. For a solid density plasma, this condition is valid even for weak fields.

Regarding the validity of the approximate one-peaked form of the structure factor (18), one can say that the first maximum of $S(q)$ is much higher than the subsequent ones unless the system is too close to the crystallization point, which corresponds to $\Gamma=170$ for the one-component plasma model [17]. In this case one, of course, should integrate Eq. (24) numerically. However, at moderate laser intensities, large momentum transfers $q>\kappa$ do not contribute (see Sec. V) and thus the one-peaked form for the structure factor is still applicable.

Let us discuss the results obtained above, in particular in comparison with the results published in the classic paper by Silin [1]. Whereas Silin treated the case of a Boltzmann plasma, the present paper deals with the case of strongly coupled plasma at an arbitrary degree of degeneracy. The main result is given by Eq. (24) and also by Eqs. (23) and (26). Formula (24) is considerably simpler than the respective result obtained by Silin [see Eq. (4.2) in [1]], and is therefore more suitable for numerical calculations as well as for asymptotic analysis.

Silin's formula contains artificially introduced cutoff parameters $q_{\max},q_{\min}$ (maximal and minimal electron momentum transfer) in order to avoid logarithmic divergencies usually emerging in the Coulomb scattering problem. Such cutoffs appear in the present paper in a natural way as the result of the mathematically consistent treatment of the divergencies: $q_{\max}=|2eE|/\omega$ corresponds to twice the quiver momentum of an electron, and $q_{\min}=\kappa$ corresponds to the location of the first maximum of the structure factor (inverse correlation length). Although the general structure of the expression for $\sigma_n(\omega)$ in our paper [see Eq. (23)] and in Silin's paper is the same, the present, more mathematically consistent analysis gives the numerical factor in Eq. (23) two times smaller than in [1].

Now let us briefly mention the connection of the nonlinear currents calculated above to bremsstrahlung absorption and harmonic generation. The rate of the energy absorption by a unit volume of a plasma can be calculated as $\langle\langle \mathbf{j}\cdot\mathbf{E}\rangle\rangle$, where $\langle\langle\ \rangle\rangle$ denotes the time averaging. It is clear from Eqs. (22) and (26) that only the first harmonic of the current j contributes to absorption, since for $n>1$ $\langle\langle\cos\omega t\cos n\omega t\rangle\rangle=0$.

The excitation of high-order harmonics of the electric current by intense laser radiation means that light of the respective frequencies can be emitted by the plasma. As we have just mentioned above, the electric current harmonics of order $n>1$ do not contribute to energy absorption and thus physically correspond to multiphoton scattering processes or harmonic generation phenomena. Let us recall (see Sec. IV) that in the high intensity limit the maximal order of harmonics generated is determined by the ratio $q_{\max}/q_{\min}$ and the amplitude of the harmonics does not depend upon n within this range. Such a "plateau" terminating at a certain highest harmonic is an attribute of harmonic generation phenomena, observed in experiments and computer simulations of atom-laser radiation interaction (e.g., [18]). This "plateau" behavior is not yet well understood. In the case of a strongly coupled plasma, considered in the present paper, a possible qualitative interpretation of the existence of the above "cutoff" can be given. Due to strong ion-ion correlation the minimum change of the electron momentum in a single scattering event is of the order of $\kappa\sim\hbar/a$, where a is the ion sphere radius. It means that the electron is effectively affected by the field of an ion at distances less than $\hbar/\kappa$ (for clearness we introduced Planck's constant $\hbar$ here). Therefore the maximal work performed by the electric field is $W\sim\hbar|eE|/\kappa$, and the electron can reemit photon of the maximal frequency $\omega'\sim W/\hbar\sim(q_{\max}/q_{\min})\omega$.

Let us return to the discussion of formulas (26) for conductivities at moderate laser intensities. It is the advantage of the strong coupling limit that all harmonics of the current can be calculated analytically using the approximation of Eq. (25). It should be emphasized that formulas (26) are not obtained by perturbation theory in terms of the parameter $eE/\omega p_F$; they are valid up to the values of unity. In the limit $E\to 0$, the first of Eqs. (26) coincides with Ohm's law in the conventional linear form. In this connection we suggest a semiempirical generalization of Eqs. (26): using experimentally measured values for the high-frequency zero-field conductivity $\sigma_0(\omega)$ we enhance the reliability of formulas (26). In the present paper, we restrict ourselves to the first and the second harmonic of the electric current, however any higher harmonic can be calculated analytically. Formulas (26) are convenient for investigating how reflection and absorption of laser radiation depend on laser intensity. They may also be useful in studying harmonics generation. Results of such studies will be published elsewhere.

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